Reply by Author to H. Guthart

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IN his comments Guthart has made a very neat derivation of the output spectrum of a sensor responsive to a plane of infinite extent and of infinitesimal thickness normal to the convection velocity of a homogenous turbulent flow. The author is familiar with this derivation through correspondence[1] with Guthart and pointed out[2] that his expression for $E_p(n)$ [see Eq. (8)] should reduce to a hot wire or "point" spectrum when $A(r)$ is a delta function. The author certainly agrees that a "point" resolution device, e.g., a hot wire probe, would respond better at high wavenumbers than the radiometer, which has comparatively poorer spatial resolution and operates as a low-pass spatial filter. This in fact was demonstrated experimentally in the author's paper[3] with regard to changes in the distance along the jet axis which the radiometer viewed (called the "aperture width," see Fig. 1 of Ref. 3). Thus the radiometer output spectrum was shown to have been processed by the low-pass filter function shown in Fig. 11 of Ref. 3.

The problem at hand is the validity of employing the hot wire spectrum as the radiometric spectrum of a thin turbulent jet cross section. Prior to studying the hot-jet radiometric data, an exact solution of the radiometric spectrum seemed necessary. However, the radiometric spectra obtained displayed a strong dependence on the radiometer aperture width (size and shape) used and in the limit of the "thin" cross section (infinitesimal aperture width w), these spectra closely resembled the hot wire temperature spectra of Corrsin and Uberoi.[4] The reduced spectral data (energy density vs wavenumber k) show the high-wavenumber spectra behavior varying from k^{-5} to $k^{-2.3}$ as the radiometric aperture width is decreased. Extrapolation of this data to an infinitesimal radiometric aperture width indicated corresponding high-wavenumber slopes behaving as k^{-2}.

While discussing these high-wavenumber slopes, it is worth pointing out that the radiometric aperture shapes (as distinguished from the width) are important. A rectangular radiometric aperture shape (such as used in Ref. 3) attenuates high wavenumbers with a k^{-1} filtering process. But any realistic aperture is rounded off (e.g., a Gaussian pulse). It "contains" relatively fewer high-frequency components than the rectangular pulse and suppresses high wavenumbers more severely (e.g., a Gaussian pulse Fourier transforms into a Gaussian pulse). Thus the very steep high wavenumber shapes k^{-5} previously referred to are due to the radiometric aperture shape and do not indicate that the hot-jet radiation from a thin slab normal to the jet axis generates a radiometric spectrum with shape k^{-4} rather than k^{-2}.

In any event, the experimental turbulent hot-jet spectra do not seem to behave as k^{-4} as suggested by Guthart. The author feels that this disparity between Guthart's results using infinite limits and experiment arises from the limited extent of the jet. The radiating jet region is not infinite in extent with respect to the jet local transverse correlation length. Generally we expect the instrument with the poorer spatial resolution to suppress the high-wavenumber radiometric spectra more than the point resolution instrument, except in the case when both instruments are viewing areas over which the spatial information is strongly correlated. In the case of radiometrically viewing a very thin slab perpendicular to the hot-jet axis, the ratio of the effective radiating cross section of the hot jet [based on Guthart's $A(r)$] to the transverse temperature correlation length becomes the parameter that indicates whether the radiometric spectrum behaves as k^{-4} when the parameter is infinite (uncorrelated flow), or as $k^{-5/3}$ when the parameter is zero (correlated flow.)

This can be shown by using Guthart's expression for $E_p(n)$, letting $A(x)$ be a delta function, normalizing lengths by a transverse temperature correlation length Λ_y, equal to longitudinal temperature correlation length Λ_{xi} and representing the upper limit in r by R

$$E_p(n) \propto \int_0^{R'} \frac{A(r')K_1\{r'[(1+S^2)]^{1/2}\}}{(1+S^2)^{1/2}} r'^2\, dr'$$

where

$(\ )' = (\ )/\Lambda$
S = reduced frequency $= 2\pi n\Lambda/\bar{U}R\Lambda = k\Lambda$
$A(x) = \delta(x)$
R = maximum jet radius of interest
$\Lambda_y = \Lambda_x = \Lambda$
$A(r') = 1$

We let the reduced frequency become very large so $S \gg 1$. Then, for $R' \gg 1$ or "uncorrelated" flow $E_p(n) \propto k^{-4}$; for $R' \ll 1$ or "correlated" flow, $E_p(n) \propto k^{-2}$.

Since the hot wire spectrum lies closer to $k^{-5/3}$ than to k^{-2}, it appears that the $k^{-2} - k^{-2.2}$ behavior observed by the author[3] for thin ($w/\Lambda \ll 1$) aperture widths very likely indicates an intermediate case where the transverse temperature correlation length Λ is on the order of the flow viewed R.

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References

[1] Guthart, H., private communication (November 18, 1966).
[2] Draper, J. S., private communication (January 10, 1967).
[3] Draper, J. S., "Infrared radiometry of turbulent flows," AIAA J. 4, 1957–1603 (1966).
[4] Corrsin, S. and Uberoi, M. S., "Spectra and diffusion in a round turbulent jet," NACA Rept. 1040 (1951).

Comment on "Ballistic Coefficients for Power Law Bodies"

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BALLISTIC coefficients of slender, axisymmetric, power law bodies of revolution were reported by Berman[1] for constant values of length, base diameter, and specific weight. The ratio of ballistic coefficient of a power law body to that of a sharp cone was computed from an empirical variation of drag coefficient with power law exponent as determined from wind-tunnel data and limited inviscid theoretical calculations. A maximum ballistic coefficient 42% larger than that of a sharp cone was determined for a power law exponent of 0.62. It would be interesting to know whether the same optimum solution could have been determined analytically from Newtonian theory.

Newtonian impact theory and Newtonian theory with the Busemann centrifugal correction give infinite drag coefficients for slender power law bodies with exponents less than or equal to $\frac{1}{2}$. Taking this as a lower limit on the exponent n, the ratio of ballistic coefficients derived from New-

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